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Ejercicio 1 (1.00 p)	Ejercicio 2 (3.00 p)	Ejercicio 3 (2.00 p)	NOTA FINAL
Ejercicio 4 (1.25 p)	Ejercicio 5 (1.25 p)	Ejercicio 6 (1.50 p)	

LOGIC EXERCISES (4 points)

[1.00p] **Exercise 1.** Propositional Language.

Encircle de well formed formulas (wffs) representing each expression. A, B, C are propositions.

Expression	fbf	fbf	fbf	fbf	fbf
It is true A and B unless it is not true C	A ^ B → ¬C	¬C → A ^ B	¬A v ¬B v ¬C	¬(A ^ B) → ¬C	(A ^ B) v ¬C
Only if it is true A but not B it is false C	¬C → A ^ ¬B	A ^ ¬B ^ ¬C	C v (A ^ ¬B)	A ^ ¬B → ¬C	(A ^ ¬B) v ¬C
For be false A and B it is sufficient that C is false	¬C → ¬(A ^ B)	¬(A ^ B) ^ ¬C	¬(A ^ B) → ¬C	C v ¬(A ^ B)	¬(¬C ^ A ^ B)
It is not necessary that A is false for B being false but not C	¬(¬B ^ C → ¬A)	¬A → ¬B ^ C	A v ¬B ^ ¬C	¬B ^ C → ¬A	B ^ ¬C ^ A

[3.00p] **Exercise 2. Argumentation.**

a) [0.50p] **Define** what is a **correct reasoning in** logic.
b) [0.75p] For the following methods,
   i) True table.
   ii) Counter-example method.
   iii) Natural Deduction.

   Explain why they proof whether a reasoning is valid and explain **how** they do it.

c) [1.75p] **Proof the validity** of the reasoning R using each of the methods in b).
   i) Reasoning R:

   P1: A v ¬B **P2**: A → B ^ C **P3**: C → ¬B

   Q: ¬B

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EXERCISES OF ALGEBRA (6 points)

[2.00p] **Exercise 3.** Systems of linear equations.

3.1. [0.25p] **Explain** the types of **elementary operations** by rows to find echelon forms in square matrices.. **Indicate** whether the following operation is elementary and of what type:

$$F1 \; <- \left(\frac{1}{2}\right)F1$$

3.2. [0.25p] Let **Ax = b** be the matricial representation of a linear system and **M** the augmented matrix. **Write** the equations of the system. Denote the unknown as $x_{i,}$ i = 1…n.

$$M = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & 1 & -2 & 1 \\ 4 & -1 & 2 & 0 \end{bmatrix}$$

3.3. [0.75p] What **features** will have **M´** it it is an **echelon form** of **M**? Calculate **M'** step by step.

3.4. [0.50p] For each elementary operation applied to calculate **M´** write its corresponding la **elementary matrix E_i**.

3.5. [0.25p] Given **M´**, is the system **consisten**? **Explain** why and write its solution if it exists.

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[1.25p] **Exercise 4**. Vector sub-spaces.

4.1. [0.50p] Determinate el **value** of **a** so that the vector **u** = **(1, a, 5)** belongs to the subspace S1 = {(1,2,3), (1,1,1)} of R^3

4.2. [0.75p] Find a **base** of the subspace S2 of R^4. **Indicate** its **dimension**.

S2 = { (1, 2,-1,3), (2, 1, 0,-2), (0,1,2,1), (3,4,1,2) }

[1.25p] **Exercise 5.** Vector spaces of a matrix.

Given the matrix **A** and its reduced matrix **A´**

$$A = \begin{bmatrix} 2 & -3 & -8 & 7 \\ 2 & -1 & 2 & -7 \\ 1 & 0 & -3 & 6 \end{bmatrix} \qquad A' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

5.1. [0.25p] What vectors of **A** form a base of the **row subspace** of **A** (*Fil A*)? Could a base of *Fil A* have a distinct number of vectors? Justify your answer.

5.2. [0.25p] Consider a base of **the column sobspace** of **A** (*Col A*), What is the maximum number of vectors it can have. Justify your answer.

5.3. [0.25p] **Calculate** two **bases** for the subspaces Fil A **and** Col A.

5.4. [0.50p] To what subspaces belong the vectors u = (3, 2) and v = (-1, 1, 1)? Justify your answer.

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[1.50p] **Exercise 6**. Eigenvalues and eigenvectors of a matrix.

6.1. [0.60p] **Choose** the correct option in each case:

a) The eigenvalues λ_i of a matrix A are the **roots** of the characteristic polynomial which is obtained from calculating the determinant:
   - a.1. det(A - λI) = 0
   - a.2. det(A + λI) = 0

b) If the **degree** of the polynomial is **3** then:
   - b.1. A has exactly 3 real eigenvalues.
   - b.2. A may have 1, 2 o 3 real eigenvalues.

c) Every eigenvalue λ_i has:
   - c.1. A finite number of eigenvectors.
   - c.2. Infinite eigenvectors associated with it.

d) Each set of eigenvectors conforms a **subspace** whose vectors:
   - d.1 Are always the vectors of the subspace *Nul A*.
   - d.2. Are the vectors of the subspace *Nul A* only for λ=0.

e) The **geometric multiplicity** is:
   - e.1 The dimension of the subspace generated by an eigenvalue.
   - e.2. The number of eigenvalues of the matrix

f) The **trace** of a matrix is coincident with:
   - f.1. The product of all the eigenvalues of the matrix.
   - f.2. The sum of all the eigenvalues of the matrix.

6.2. [0.90p] Given the matrix B

$$B = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ 2 & 2 & a \end{bmatrix}$$

6.2.1 **Proof** that the vector $v = (1, -1, 0)$ is an **eigenvector** de B associated with the eigenvalue $\lambda = 2$.

6.2.2 **Calculate** the value of $\mathbf{a}$ so that the **eigenvalue** $\lambda = 2$ is the **unique** eigenvalue of B.